Admissibility of the Spectral Band-Limiter τ_B Post-Ω_{4b} Selection

Abstract

We formalize and validate the admissibility of the spectral band-limiter operator τ_B within the UNNS substrate, when applied after canonical Ω_{4b} selection. Admissibility is established through empirical contraction of residual error, preservation of protected macro-invariants, and robustness across parameter variation. This constitutes the first confirmed $\Omega \to \tau$ coupling in the UNNS operator pipeline.

1 Preliminaries

Let E denote an ensemble of graph-structured objects with fixed node count $n = 32$ and ensemble size $M = 100$, generated under a common stochastic generator. Let Ω_{4b} denote the canonical Ω-selection operator with fixed keep fraction $f = 0.3$, producing a filtered ensemble $\Omega_{4b}(E)$.

Let $R_\Lambda(\cdot)$ denote the residual functional measuring deviation from the target invariant manifold defined by the canonical V-median of the ensemble.

2 Definition (Admissibility)

A τ-operator is said to be *admissible post-*Ω_{4b} if, when applied to $\Omega_{4b}(E)$, it satisfies the following conditions:

1. **Residual contraction:**

$$R_\Lambda(\tau(\Omega_{4b}(E))) < R_\Lambda(\Omega_{4b}(E)) - \Delta$$

   for a fixed contraction threshold $\Delta > 0$.

2. **Contraction ratio:**

$$\mathrm{CR} = \frac{R_\Lambda(\tau(\Omega_{4b}(E)))}{R_\Lambda(\Omega_{4b}(E))} < 1.$$

3. **Macro-invariant preservation:** All protected macro-invariants exhibit relative drift below a fixed tolerance δ.
4. **Acceptance stability:** The acceptance rate of Ω_{4b} remains unchanged under τ.

3 Theorem (Admissibility of τ_B)

Theorem. The spectral band-limiter operator τ_B is admissible post-Ω_{4b} in the sense of Definition 2 for $n = 32$, $M = 100$, and parameters

$$T = 10, \quad \lambda \in [0.01, 0.05], \quad \mu = 0.02,$$

with admissibility thresholds $\Delta \geq 0.002$ and $\delta = 0.05$.

4 Proof (Empirical)

The proof proceeds by direct evaluation of the admissibility criteria.

Residual contraction

Across multiple independent runs with fixed ensemble size and varying random seeds, the following was observed:

$$R_\Lambda(\Omega_{4b}(E)) \approx 3.2 \times 10^{-3}, \quad R_\Lambda(\tau_B(\Omega_{4b}(E))) \approx 1.1 \times 10^{-4}.$$

Hence,

$$R_\Lambda(\Omega_{4b}(E)) - R_\Lambda(\tau_B(\Omega_{4b}(E))) \approx 3.1 \times 10^{-3} > \Delta,$$

and the contraction ratio satisfies $\mathrm{CR} \ll 1$.

Macro-invariant preservation

The following protected macro-invariants were monitored:

- Scaled spectral radius
- Energy per node
- Degree entropy

In all admissible runs, the maximum observed invariant drift was below 2.7%, well within the tolerance $\delta = 5\%$.

Stability and robustness

Admissibility of τ_B was confirmed for multiple values of λ in the range $[0.01, 0.05]$ and across distinct random seeds, demonstrating robustness rather than parameter fine-tuning.

Acceptance consistency

The Ω_{4b} acceptance rate remained fixed at 30% in all runs, indicating that τ_B does not retroactively bias Ω-selection.

Conclusion

All admissibility criteria are satisfied. Therefore, τ_B is admissible post-Ω_{4b} within the stated regime.

□

5 Discussion

This result establishes the first validated $\Omega \to \tau$ coupling in the UNNS substrate. Importantly, admissibility of τ_B emerges only after Ω_{4b} selection, confirming the stratified operator hypothesis: τ-operators act as stabilizers only on pre-filtered ensembles.

Note on reproducibility and seeding. All results reported in this work were revalidated under a precision-safe seeding mechanism. Input seeds are parsed deterministically and mapped to a fixed 32-bit internal state, eliminating floating-point aliasing for large seed values. Admissibility of the spectral band-limiter τ_B post-Ω_{4b} selection was confirmed to persist under this correction.

Extension of admissibility beyond the present ensemble size, generator class, and Ω-selection variant remains an open problem.

6 Implications

The admissibility of τ_B provides a canonical reference τ for:

- Operator benchmarking
- Parameter regime exploration

- Comparative analysis with multi-scale and curvature-equalizing τ families

This marks the transition from exploratory τ-testing to structured operator theory within the UNNS framework.